A Quantitative Framework for Life Insurance Liquidity Risk Management

I sketch a very high-level outline of an all-inclusive liquidity risk management Linear Program (LP) optimization problem that could be automatically solved daily to manage the General Account (GA) portfolio of a life insurer. It incorporates actuarial modeling for death claims, dynamic lapse, policy loans and the daily net premium inflows. I emphasize the focus is not statutory-type actuarial reserving – I instead outline a general-purpose mathematical framework for operational capital allocation to meet the firm's liquidity requirements under baseline and stressed scenarios.

Scenario Design

In the life insurance context, the Liquidity Coverage Ratio (LCR) is the ratio of the net cash demand conditional on $scenario - s$, C_s, to the General Account asset portfolio value, W_s-

$$L_s^* = \frac{C_s}{W_s}$$

The net cash demand for scenario-s is -

$$C_s = \underbrace{D_s}_{\substack{death\\claims}} + \underbrace{L_s}_{\substack{dynamic\\lapse}} + \underbrace{P_s}_{\substack{policy\\loans}} - \underbrace{I_s}_{\substack{net\ premium\\inflows}}$$

Each $scenario - s$ is defined by macroeconomic variable forecasts (e.g., Federal Reserve DFAST scenarios, NAIC actuarial scenarios, firm-specific macroeconomic forecasts, etc.) and the actuarial assumptions that drive them. For liquidity risk management it is reasonable to have multiple scenarios (e.g., baseline, adverse rates, severely adverse equity market, etc.) over different time horizons (e.g., overnight, one week, one month, one quarter, etc.). Implicit in the definition for LCR and net cash demand is the explicit linkage between the policy portfolio and the asset portfolio backing these liabilities.

The LP Optimization Problem

Consider a General Account portfolio allocated across N fixed-income or credit asset classes or security positions (e.g., Treasuries, agency MBS, IG corporates, HY corporates, private credit, alternatives, etc.). The portfolio weight vector is $\boldsymbol{w}_s = [w_0\ w_1 \dots w_{N-1}]^T$. For each asset $class - i$ and $scenario - s$ define -

- $y_{i,s}$ = yield spread over risk-free.
- $\lambda_{i,s} \in [0,1]$ = liquidation fraction – share of $w_{i,s}W_s$ that is recoverable within a stress liquidation horizon T_s at market depth, net haircut $h_{i,s}$
- $W_s =$ total portfolio market value

All variables are conditioned on the $scenario - s$. In order to lighten the notation I will now suppress the subscript-s.

Hence the liquid value of asset $class - i$ under normal conditions is –

$$\ell_i = \lambda_i(1 - h_i)w_i W$$

The core LP optimization problem is to maximize the asset portfolio yield –

$$\max_{\boldsymbol{w}} \boldsymbol{w}^T \boldsymbol{y}$$

subject to the following constraints:

Liquidity constraint	$\boldsymbol{\lambda}^T \boldsymbol{w} \geq L^*$
Budget constraint	$\boldsymbol{w}^T \mathbf{1} = 1$
No shorting allowed	$w_i \geq 0, \forall i$
Risk and regulatory constraints	$\boldsymbol{A}\boldsymbol{w} \leq \boldsymbol{b}$

where L^* is the **minimum required liquidity coverage ratio** derived from the scenario design framework, and the $\boldsymbol{A}\boldsymbol{w} \leq \boldsymbol{b}$ constraint captures duration limits, concentration limits, credit quality floors, etc. $\mathbf{1}$ is the unit normal vector. This is an LP problem in $\boldsymbol{w}$ (given fixed $\boldsymbol{y}$, $\boldsymbol{\lambda}$). There are multiple software libraries available to solve the system, e.g., IMSL, Python SciPy and PuLP.

A few comments on the model parameters are in order. The liquidation fractions λ_i are scenario dependent, hence they should also have a scenario subscript, $\lambda_{i,s}$. They are highly specific to the insurance firm and its dealer relationships. A simple proxy may be the ratio of historical execution versus mid-market prices across asset classes. Likewise, the yield spreads y_i and haircuts h_i are scenario dependent and specific to the insurance firm and its dealer relationships.

Rodney Greene, PhD, is the President of Martingale Solution Group, a boutique consulting firm specializing in the financial services industry. We apply our expertise in model validation and bank regulatory compliance to tackle our clients' toughest risk management challenges. We cover both AI/ML and non-AI/ML model risk cases. Martingale Solution Group is certified by the National Veteran-Owned Business Association (NaVOBA). Contact us for a consultation.

214-797-1630
Rodney.Greene@martingalesg.com